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Integrality of the Monopole Number in SU(2) Yang-Mills-Higgs Theory on $\mathbb{R}^3$ *

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Abstract. We prove that in classical SU(2) Yang-Mills-Higgs theories on $\mathbb{R}^3$ with a Higgs field in the adjoint representation, an integer-valued monopole number (magnetic charge) is canonically defined for any finite-action $L^2_{1,\mathrm{loc}}$ configuration. In particular the result is true for smooth configurations. The monopole number is shown to decompose the configuration space into path components.

Introduction

In classical Yang-Mills-Higgs theories over a Riemannian manifold M, one fixes a principal bundle

$$\begin{array}{c} P \\ \downarrow{\scriptstyle G} \\ M \end{array}$$

and studies the *action functional*

$$a(A,\Phi)=\tfrac{1}{2}\int_M (|F_A|^2+|d_A\Phi|^2). \tag{1}$$

This functional is defined on the *configuration space*

$$\mathscr{C}=\{(A,\Phi)\in\mathscr{A}\times\mathscr{E}\,|\,a(A,\Phi)<\infty\}, \tag{2}$$

where $\mathscr{A}$ is the space of $L^2_{1,\mathrm{loc}}$ connections on P, and $\mathscr{E}$ is the space of $L^2_{1,\mathrm{loc}}$ sections of some (fixed) associated vector bundle E. (This is the most general configuration space. Often one considers only *smooth* connections A and sections Φ, but for many applications this restriction is inconvenient. If a potential term is included in the action, the configuration space again is smaller.) The symbols F_A, $d_A\Phi$ denote the curvature of $A\in\mathscr{A}$ and the covariant derivative of $\Phi\in\mathscr{E}$, respectively; norms

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