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# The Broadwell Model for Initial Values in $L^1_+(\mathbb{R})$

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**Abstract.** The Cauchy problem for the Broadwell model is shown to have a global mild solution for initial data in $L^1_+(\mathbb{R})$ with small $L^1$-norm, and a local solution for arbitrary initial data in $L^1_+(\mathbb{R})$. For data which are small in $L^1(\mathbb{R})$, the asymptotic behaviour of the solutions as $t \to \infty$ is determined. Moreover, it is shown that a global solution exists for all initial values in $L^1_+(\mathbb{R})$ with finite entropy if the $H$-Theorem holds.

## Introduction

The Broadwell model is one of the simplest non-trivial discrete velocity models of the Boltzmann equation and has found a lot of attention as a model problem in kinetic theory. Nishida and Mimura [5] and Crandall and Tartar [6] have shown that the Cauchy problem has always a global unique solution for bounded initial data. Inoue and Nishida have studied the asymptotic stability of equilibrium solutions [3], and Caflisch and Papanicolaou have investigated the fluid dynamical limit as the mean free path between collisions tends to 0 [1]. Open questions are the asymptotic behaviour of solutions as $t \to \infty$ and the largest possible class of admissible initial values.

The last question is of relevance because the mass conservation law suggests $L^1$-spaces as natural spaces in which to look for solutions of the Boltzmann equation. However, the quadratic terms in the collision operator are a priori not defined in $L^1$, and this is one major reason why existence theorems for the Boltzmann equation have only been proven for smaller sets of initial data and in general only locally in time.

In this paper I investigate the question of solvability of the Broadwell model for initial values in $L^1_+(\mathbb{R})$. The basic idea is to obtain an a priori upper bound of the solution by solving a model with suitably truncated collision terms, for which I give monotone approximations – this is done in Sect. 2 for nonnegative initial data

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