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On the Structure of Tensor Operators in SU3*

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Abstract. A global algebraic formulation of SU3 tensor operator structure is achieved. A single irreducible unitary representation (irrep), V, of $\mathfrak{so}(6,2)$ is constructed which contains every SU3 irrep precisely once. An algebra of polynomial differential operators $\mathscr{A}$ acting on V is given. The algebra $\mathscr{A}$ is shown to consist of linear combinations of all SU3 tensor operators with polynomial invariant operators as coefficients. By carrying out an analysis of $\mathscr{A}$, the multiplicity problem for SU3 tensor operators is resolved.

1. Introduction

The theory of tensor operators has been fully developed only for the symmetry group SU2—the quantal angular momentum group—and the resulting theory is of fundamental importance in almost all applications of angular momentum in physics [1]. It is to be expected that the development of an analogous theory of tensor operators for the symmetry group SU3 might possibly be of comparable importance since this symmetry—$\mathrm{SU}^{\mathrm{color}}$—is held to be exact and of fundamental importance in hadronic physics; a distinct SU3 symmetry is known to be of practical importance as an approximate symmetry of the nuclear shell model.

Quite early in the development of the theory of tensor operators, Wigner [2] achieved a classification of those symmetry groups for which a direct analog to the SU2 tensor operator construction was possible. Such groups were termed *simply reducible*, and there are two conditions: the group must be (a) ambivalent (g and g^{-1} belong to the same class) and (b) multiplicity free (in the reduction of the Kronecker product of two irreps, no irrep occurs more than once). The group SU3 fails both criteria; it is neither ambivalent nor multiplicity-free. Mackey [3], however, showed that ambivalence could be weakened to *quasi-ambivalence* (the group possesses an involutive anti-automorphism). SU3 is indeed quasi-ambivalent [4] (using conjugation as the involution), but the problem of multiplicity is far more difficult to resolve.

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