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A Note on $D(k, 0)$ Killing Spinors

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Abstract. The equations for the $D(k, 0)$ Killing spinor fields are integrated assuming that the left conformal curvature does not vanish and that either $k \neq 2, 4, 6, \ldots$, or the Einstein vacuum field equations are satisfied.

1. Introduction

In a remarkable paper, Walker and Penrose [1] showed that every type D solution of the Einstein vacuum field equations admits a quadratic first integral of the null geodesic equations. Their result, later generalized by Hughston et al. [2] to a class of type D solutions of the Einstein-Maxwell equations, is based on the existence of a Killing spinor, from which a conformal Killing tensor of valence two is constructed. The proof given by Walker and Penrose follows from the Bianchi identities and provides a method to find explicitly the above mentioned conformal Killing tensor.

The equations for the Killing spinors have been studied by Hacyan and Plebañski [3] in the context of complex Riemannian geometry, which contains the case of real spacetimes. A direct integration of the equations for Killing spinors of type $D(k, 0)$ has been done by Finley and Plebañski [4] in the case of $\mathscr{H}$ spaces (left-flat spaces). In the present work the equations fort Killing spinors of type $D(k, 0)$ are integrated under some restrictions. The results apply to complexified space times as well as to real ones. The formalism and notation used here follow those of Plebañski [5]. All the spinorial indices are manipulated according to the convention $\psi_A = \varepsilon_{AB}\psi^B$, $\psi^A = \psi_B \varepsilon^{BA}$, and similarly for dotted indices.

2. Integrability Conditions

Let $L_{AB\ldots D}$ be a $D(k, 0)$ Killing spinor [1], that is, $L_{AB\ldots D}$ is a totally symmetric spinor field with $2k$ indices that satisfies the equation[1]

$$\nabla^{\dot{S}}_{(R} L_{AB\ldots D)} = 0. \tag{1}$$

1 Round brackets denote symmetrization of the indices enclosed