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Generalized Internal Long Waves Equations: Construction, Hamiltonian Structure, and Conservation Laws

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Abstract. A general class of the ILW type equations is constructed. We introduce a Hamiltonian structure and construct an infinite number of conservation laws.

0. Introduction

Recent studies [1–4] have shown that the following equation

$$u_t+\delta^{-1}u_x+2uu_x+T[u_{xx}]=0, \tag{1}$$

where $T[u](x)=\int\limits_{-\infty}^{+\infty}(1/2\delta)\coth(\pi(y-x)/(2\delta))u(y)dy$ is of mathematical and physical interest. This equation has many important mathematical features similar to those of the Korteweg de Vries equation (KdV). Physically it represents internal long waves (ILW) in a stratified fluid of finite depth characterized by the real parameter δ [5–6]. The limiting cases of ILW are: the KdV ($\delta\to 0$) and the Benjamin-Ono (BO) ($\delta\to\infty$) equations [3].

For a long time the outward similarity of the ILW (BO) equation with the KdV equation, the existence of an infinite number of conservation laws, a Bäcklund transformation and so on have suggested that there should be a general theory where ILW (BO) would be the simplest example (like KdV for the general Lax equations). As early as October 1978 at the Leningrad Soliton Conference L. D. Faddeev emphasized the importance of studying the BO equation by making use of group-theoretical methods, as with KdV.

In this paper we consider some aspects of the theory of the ILW type equations. Let us state the main results of this paper.

The first result is the construction of a general class of the ILW type equations by means of the formal Zakharov-Shabat "dressing" method [7] (Zakharov-Shabat's technique for ILW was discovered in [4]). Let L_0 be the symbol of a skew Hermitian differential operator with constant coefficients, and let K be the symbol of a Volterra operator with the coefficients holomorphic in the strip,