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A New Proof of the Existence and Nontriviality of the Continuum φ_2^4 and φ_3^4 Quantum Field Theories

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Abstract. We use Schwinger-Dyson equations combined with rigorous "perturbation-theoretic" correlation inequalities to give a new and extremely simple proof of the existence and nontriviality of the weakly-coupled continuum φ_2^4 and φ_3^4 quantum field theories, constructed as subsequence limits of lattice theories. We prove an asymptotic expansion to order λ or λ^2 for the correlation functions and for the mass gap. All Osterwalder-Schrader axioms are satisfied except perhaps Euclidean (rotation) invariance.

1. Introduction

The proof of existence of the superrenormalizable φ_3^4 quantum field theory along with the analysis of some of its physical properties (mass gap, particle structure, symmetry breaking...) is one of the grand achievements of the Constructive Quantum Field Theory program. We direct the reader to [1, 2] and to the references cited in [2, 3] for background. Even a casual inspection of that literature will reveal how difficult and clever are the methods invented and used by previous workers on φ_3^4.

In this paper we present a novel – and, we believe, extremely simple – approach to the φ_3^4 quantum field model. We have tried hard to make our presentation comprehensible to experts and non-experts alike. We therefore beg the expert's indulgence as we review some well-known facts. We reassure the non-expert that any technical terms used in this Introduction will be defined in an accessible manner in the main body of the paper.

We begin with a brief summary of our methods, since these are possibly more interesting than our results. Indeed, *all* of our results are known ones; what was previously unknown was that they could be obtained so easily.

Theoretical physicists have long made use of a system of coupled non-linear integral equations, called the Schwinger-Dyson equations (or "field equations"), in

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