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Potentials for the Supersymmetric Nonlinear σ-Model

Luis Alvarez-Gaumé[1,*] and Daniel Z. Freedman[2,**]

1 Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA
2 Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Abstract. The most general structure for potential terms compatible with $N=1$, $N=2$, and $N=4$ supersymmetry in the nonlinear σ-model in two space-time dimensions is determined. The differential geometry of the internal manifold of the model plays an important role in the method used and in the results. An interesting application of nontrivial dimensional reduction is found.

1. Introduction

A strong connection has been established between extended supersymmetry in the nonlinear σ-model in two space-time dimensions and the differential geometry of the internal manifold M on which the model is defined. $N=2$ supersymmetry requires that M is a Kahler manifold [1, 2], $N=3$ supersymmetry implies $N=4$, and $N=4$ requires that M is hyperKahler. This connection between complex differential geometry and supersymmetry strongly constrains renormalization counterterms [3, 4], and there are strong indications that at least the $N=4$ theories are ultraviolet finite to all orders in perturbation theory [2, 3].

In this paper we consider the inclusion of potential terms with a coupling constant of the dimensions of mass in the model. One motivation for this arises from the infrared problems of massless scalars in two dimensions. In the $O(n)$ and CP^n models there is spontaneous generation of mass [5, 6] due to asymptotic freedom, but the resolution of the infrared difficulty is unclear for $N=4$ models which are ultraviolet finite. The potential gives massive excitations at the classical level which circumvents the infrared problems.

In the bosonic σ-model the potential $V(\phi)$ can be an arbitrary function on M. In $N=1$ supersymmetry one can add an arbitrary superpotential $W(\phi)$, as is well known, but additional parity non-conserving terms are possible if M possesses

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