Commun Math Phys 90, 461–472 (1983)

Communications in Mathematical Physics

The Group Theoretical Aspects of Infinitesimal Riemann–Hilbert Transform and Hidden Symmetry

Wu Yong-Shi*,**

Centre de Physique Theorique, C.N.R.S.-Luminy, Case 907, F-13288 Marseille Cedex 9, France and Max Planck Institut für Physik and Astrophysik, D-8000 München 40, Federal Republic of Germany

Abstract. We obtain explicit expressions for infinitesimal regular Riemann–Hilbert (RH) transforms. Using them, the group theoretical aspects of infinitesimal RH transforms are discussed with an eye to the comparison with the hidden symmetry transformations proposed by us before. We find that the RH transforms have very rich group structure; e.g. in the 2-d principal chiral models, their group contains two Kac–Moody algebras as subalgebras. But not all of them are nontrivial hidden symmetries of the theory.

I. Introduction

Very recently there has been much progress in understanding the existence and Lie-algebraic structure of an infinite-parameter hidden symmetry in 2-d principal chiral models [1–7] and 4-d self-dual Yang–Mills equations [8–12]. Basically, there are two approaches to this problem. One is explicit construction of the infinitesimal hidden symmetry transformations for the basic fields under consideration [1–5, 7] and [9, 11, 12]. Another one is to use the regular infinitesimal Riemann–Hilbert (RH) transform for the auxiliary quantities—the solutions to the linearization equations [6 and 10]. The original motivation of this paper was to investigate the connection between these two approaches.

However, during the course of this study, it gradually became clear that our explicit expressions for infinitesimal RH transforms and the evaluation of their commutators are in fact generally true. Therefore the study has developed into a general discussion of the group theoretical aspects of (infinitesimal) RH transforms which have been already a powerful tool for solving many nonlinear equations. But we prefer to start with a concrete example, in order to make the presentation not too abstract at the beginning.

* Institute for Advanced Study, Princeton, NJ 08540, USA

** Permanent address: Institute of Theoretical Physics, Academia Sinica, Box 2735, Beijing, People's Republic of China