# A Property of the Structure Constants
# of Finite Dimensional Compact Simple Lie Algebras

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**Abstract.** We consider products of structure constants of a finite-dimensional compact simple Lie algebra, in which all indices except a few are contracted in pairs. We prove that such a product is zero if only one index is free, is proportional to the Cartan-Killing tensor if two indices are free and is proportional to a structure constant itself if three indices are free. For SU($n$), $n \geq 3$ we also consider products of usual $d$ (related to the anti-commutator) and structure constants $f$. The results for one and two free indices are still valid. For three free indices the product is proportional to either an $f$ or a $d$ according to whether the number of $f$'s in the product is odd or even.

## 1. Introduction

In quantum chromodynamics (QCD), the various fields carry colour and the interaction vertices depend explicitly on quantities connected with the colour group SU(3)$_c$, which is gauged to produce the interactions. In particular, the colour dependence of the bare triple gluon and ghost-gluon vertices is given by the structure constants $c_{ijk}$ of the colour group (forgetting for the moment about the contravariant or covariant character of the indices). The question then arises about the colour dependence of the vertices in higher loop orders. Is it still given by $c_{ijk}$ as above, or are there additional terms with a different dependence on the colour indices? This question is important from the viewpoint of having a renormalizable theory. The point is that unless the counterterms required for these vertices in higher orders depend only on $c_{ijk}$, such theories will not be renormalizable in the usual way. Fortunately, using charge conjugation invariance, one can show [1] that the proper triple gluon vertex depends solely on $c_{ijk}$. However, this proof does not work for the ghost-gluon vertex. Instead one can use the Slavnov-Taylor identity [2] connecting the two vertices to show [1] that in two momentum subtraction schemes [3] the divergent part of the ghost-gluon vertex depends only on $c_{ijk}$, though finite parts with a different colour dependence are not ruled out. Much before this proof was found, one was led to speculate that there may be a