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On Perturbation Theory for Regularized Determinants of Differential Operators

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Abstract. A perturbation theory for determinants of differential operators regularized through the ζ-function technique is presented. The application of this approach to the study of chiral changes in the fermionic path-integral variables is discussed.

1. Introduction

In the Feynman path-integral approach to quantum theory one is naturally led to the computation of determinants of differential operators. These determinants clearly diverge because the eigenvalues λ_j increase without bound. Therefore, it is necessary to adopt some regularization procedure. One technique which has proved to be very useful is the ζ-function regularization [1]. When A is an elliptic invertible operator of order $m>0$, defined on some compact manifold M without boundary of dimension n, one forms a generalized ζ-function from the eigenvalues λ_j of A:

$$\zeta(s,A)=\sum_j \lambda_j^{-s}. \tag{1.1}$$

This series converges only for $\operatorname{Re}s>n/m$, but $\zeta(s,A)$ can be analytically extended to a meromorphic function of s in the whole complex plane [2]. In particular it is regular at $s=0$. The derivative of the ζ-function at $s=0$ is formally equal to $-\sum_j \log\lambda_j$. One can therefore define the regularized determinant of A, $\operatorname{Det}(A)$, to be $\exp(-d\zeta/ds)|_{s=0}$.

The purpose of this paper is to study the behavior of $\operatorname{Det}(A)$ when the operator A is perturbed by another operator of smaller order. To be more precise, we are going to prove in Sect. 2 that

$$\operatorname{Det}\left(A+\sum_{j=1}^{p}\varepsilon^j A_j\right)=\operatorname{Det}(A)\exp[\varepsilon c_1+\varepsilon^2 c_2+\ldots+\varepsilon^r c_r+O(\varepsilon^{r+1})] \tag{1.2}$$

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