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Fermions without Spinors

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Abstract. The classical Kähler equation for an inhomogeneous differential form is analysed in some detail with respect to the physical properties of its Minkowski space solutions. Although the components of the field contain only integer representations of the Lorentz group for a physical interpretation of the quantum theory, we impose fermionic commutators. The electromagnetic interactions are identical to those of a Dirac spinor field with an extra fourfold degeneracy. Possibilities for the interpretation of the extra degrees of freedom are discussed.

Introduction

This paper is concerned with a field system possessing certain remarkable properties. Some of these properties will be analysed mathematically and an attempt made to relate them to physical properties shared by fermions and bosons. The dynamics of the field system to be considered was first postulated by Kähler [1] in 1961 although there is some evidence to suggest that Darwin was aware of a similar description [2] in 1928. Apart from one notable exception [3] these ideas appear to have received little further attention. In view of the current enthusiasm for multicomponent field systems describing bosons and fermions and because of its natural formulation in arbitrary pseudo-Riemannian spaces, we feel that this system deserves further investigation.

It is notable that the classical field theories underlying all modern descriptions of the fundamental interactions between elementary particles can be formulated in terms of differential forms on space-time; i.e. tensors with antisymmetric components. This reflects the property that they can be derived from an action principle which may be globalised with the aid of differential forms. Internal degrees of freedom associated with the particles may be incorporated by using forms that take values in appropriate vector spaces, particularly Lie algebras.

When the Stern-Gerlach experiment was interpreted in terms of the intrinsic spin of the electron, this degree of freedom was also coded into an internal state