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Asymptotic Observables on Scattering States

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Abstract. In quantum mechanical potential scattering theory we use selected observables to describe the asymptotic properties of scattering states for long times. E.g., we show for the position and momentum operators that for $\Psi \in \mathscr{H}^{\mathrm{cont}}(H)$,

$$\left(m\frac{\mathbf{x}}{t} - p\right)e^{-iHt}\Psi \to 0,$$

and that the set of outgoing states is absorbing. This is obtained easily without any detailed analysis of the interacting time evolution. The class of forces includes highly singular and very long range potentials.

The results may serve as an intermediate step in a proof of asymptotic completeness; as a particular application we present a simple proof of completeness for Coulomb systems.

I. Introduction

In potential scattering theory the states in the continuous spectral subspace of the Hamiltonian are well known to have a simple time evolution asymptotically if the perturbation is suitably localized. If the potential is of short range, then the free time evolution is a good approximation of the interacting one in the far future and in the remote past. Similarly a modified free time evolution can be used if long-range forces are present. This fact, called asymptotic completeness, allows one to deduce various properties of the asymptotic motion since free or modified free time evolutions can be controlled easily. E.g., the position and the momentum vectors become parallel at large times and have been antiparallel in the remote past.

Sometimes one is interested in obtaining partial information about a state without first proving asymptotic completeness, i.e. by studying the interacting time evolution itself. In particular we think of two reasons for doing this. In the case of very long range forces it may be hard to construct and control a modified free time evolution or its existence may be unknown (e.g. if $V(x) \sim [\ln(\ln|x|)]^{-1}$ as $|x| \to \infty$). Nevertheless physical intuition suggests that for the position and momentum of a