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Non-Gaussian Fixed Points of the Block Spin Transformation. Hierarchical Model Approximation

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Abstract. With the use of analyticity techniques recently developed by the authors, the ε- and $\frac{1}{N}$-expansion type arguments are turned into a rigorous control of the non-Gaussian fixed point of the hierarchical model renormalization group. The present approach should extend beyond the hierarchical approximation and result in mathematical theory of the critical point of statistical mechanics or quantum field theory in three dimensions for small ε or large N.

1. Introduction

The present paper is the first step in the study of the critical point of the classical statistical mechanical systems with non-Gaussian long distance behaviour and of its scaling limit. As a presumed example of such a system one may consider a lattice model with the Gibbs state given formally by

$$\frac{1}{Z}\exp\Big[-\sum_x v(\phi_x)\Big]d\mu_G(\phi), \tag{1}$$

where the spin variables $\phi_x \in \mathbb{R}^N$, $x \in \mathbb{Z}^d$, $d\mu_G$ is the Gaussian measure with mean zero and covariance $G \equiv (G_{xy})$ with $|G_{xy}| \sim |x-y|^{-\alpha}$ for large $|x-y|$, $\alpha < \frac{1}{2}d$, and where, for example, $v(\phi) = \frac{1}{2}m^2\phi^2 + \lambda(\phi^2)^2$ with $m^2 \in \mathbb{R}^1$, $\lambda > 0$. The understanding of the behavior of (1) for the critical value of m^2 (where the correlation length becomes infinite) is based on the renormalization group (RG) self-similarity idea [1, 11, 14, 15, 20]. Under RG transformations which integrate out successively the short range degrees of freedom, a critical system should go to a fixed point. In two situations: when $\varepsilon \equiv \frac{1}{2}d - \alpha$ or when $\frac{1}{N}$ are small, the RG transformation may be computed perturbatively as a formal power series in ε or in closed form when

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