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# Probability Distribution of Random Paths in the Ising Model at Low Temperature

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**Abstract.** The random walk representation of the $n$-dimensional Ising model exhibits the 2-point correlation function $\langle\sigma(x)\,\sigma(y)\rangle$ as a sum of positive contributions of paths $\omega$ from $x$ to $y$. We derive upper bounds on the individual terms in this sum for low temperatures. Each term tends to zero as $\beta\to\infty$, while the correlation function itself tends to 1. Therefore increasingly more and longer paths contribute when $\beta$ is lowered.

## 1. Introduction

This investigation was motivated by the Durhuus-Fröhlich [random surface] representation [1] of the Wilson loop in lattice gauge theories: The expectation value of the Wilson loop in an $n$-dimensional theory can be written as an average of the product of correlation functions of $(n-1)$-dimensional spin systems with fluctuating couplings. For each of these correlation functions one uses a random path representation of the kind that is investigated in the present paper. These paths lie in hyperplanes above each other and vary independently. They can be combined to a surface which has the Wilson loop as its boundary. The question arises what surfaces will make a nonnegligible contribution to the Wilson loop at a given temperature [coupling constant]. For instance one will expect that the confining (high temperature) phase and the Higgs (low temperature) phase of a pure $\mathbb{Z}_2$ lattice gauge theory in 3 or 4 dimensions [2] will be characterized by a qualitatively different probability distribution of these random surfaces (compare [3]).

In this paper the corresponding question for the correlation function of the Ising model is investigated. A qualitatively different behavior at high and low temperatures is found. At high temperatures it is immediately seen from the random path representation that the shortest path makes the dominant contribution. The main problem was therefore to analyse the multiple integrals that appear in the random path representation at low temperatures. This problem was solved with the help of a combination of high temperature (Mayer-) and low