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On Multimeron Solutions of the Yang-Mills Equation

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Abstract. We study a singular boundary value problem introduced by Glimm and Jaffe for the purpose of obtaining solutions of the Euclidean Yang-Mills equations with isolated singularities along an axis. Using comparison techniques, we prove existence, asymptotic behavior and also uniqueness in some special cases.

1. Introduction

In this note we study solutions of the singular boundary value problem

$$Lu \equiv \Delta u + \frac{1}{x_1^2}(u-u^3)=0 \quad \text{in} \quad R_+^2, \tag{1.1}$$

$$u(0,x_2)=(-1)^j, \qquad a_j<x_2<a_{j+1}, \qquad j=0 \quad \text{to} \quad 2n, \tag{1.2}$$

where $R_+^2=\{x=(x_1x_2)\in R^2 : x_1>0\}$ and

$$-\infty=a_0<a_1<\ldots<a_{2n}<a_{2n+1}=+\infty. \tag{1.3}$$

The boundary value problem (1.1), (1.2) was introduced by Glimm and Jaffe [4] for the purpose of obtaining solutions of the Euclidean Yang-Mills equations in R^4 with isolated singularities along an axis. The existence of a solution was obtained heuristically in [4] and established in a rigorous way using variational methods in [6]. The solutions of the Yang-Mills equations that arise from (1.1), (1.2) are known as *multimeron* solutions and may be thought of as describing "pseudoparticles" located at the singular points (see [5]).

The two-meron solution ($n=1$) of (1.1), (1.2) is given explicitly by the formula

$$u(x)=\frac{(x-a_2e_2)}{|x-a_2e_2|}\cdot\frac{(x-a_1e_2)}{|x-a_1e_2|}, \tag{1.4}$$

(where e_1, e_2 is the standard basis of R^2). Here we exploit this fact and the invariance of Eq. (1.1) under change of scale $u(x)\to u(\lambda x)$ and inversions about a

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