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Translation Invariant Equilibrium States of Ferromagnetic Abelian Lattice Systems

Charles-Edouard Pfister

Département de Mathématiques, Ecole Polytechnique Fédérale, 61, Av. de Cour, CH-1007 Lausanne, Suisse

Abstract. The structure of the set of all translation invariant equilibrium states is determined for all temperatures, for which the free energy is differentiable. Models with several phase transitions are discussed rigorously.

1. Introduction

The class of ferromagnetic models considered in this paper includes the Ising model, XY or Rotator model, Ashkin-Teller model, $\mathbb{Z}_n$-model, Potts model and so on. For any temperature one can construct a translation invariant equilibrium state, $\langle\cdot\rangle^0$, which is also an extremal equilibrium state (see Sect. 3). Moreover, if S_0 is the internal symmetry group (see Sect. 4) and $S(\beta)$ the subgroup of S_0, which leaves $\langle\cdot\rangle^0$ invariant, then *all* equilibrium states are $S(\beta)$-invariant. In particular, when $S_0 = S(\beta)$, there is no symmetry breakdown of S_0. On the other hand, when $S(\beta_0) \neq S_0$, all equilibrium states are $S(\beta_0)$-invariant and therefore there is a natural action of the quotient group $S_0/S(\beta_0)$ on the equilibrium states. Using this action on $\langle\cdot\rangle^0$, one obtains new extremal equilibrium states $\langle\cdot\rangle^\theta$, $\theta \in S_0/S(\beta_0)$. Let λ be any probability measure on $S_0/S(\beta_0)$, which is translation invariant (with respect to the action of $\mathbb{Z}^d$ on $S_0/S(\beta_0)$). The state

$$\int_{S_0/S(\beta_0)} \lambda(d\theta)\langle\cdot\rangle^\theta \tag{1.1}$$

is clearly a translation invariant equilibrium state. The main result of Sect. 4 ensures, that all translation invariant equilibrium states at inverse temperature β_0 are given by (1.1), if and only if the free energy is differentiable with respect to β at β_0. This result was already known for Ising ferromagnetic systems by the works of Slawny [1] Lebowitz [2], [3], and Bricmont, Lebowitz, Pfister [4]. The main technical tool is correlation inequalities [5], which are derived using ideas of Ginibre in his basic paper on correlation inequalities [6]. One obtains in this way results similar to those of Lebowitz in [2].

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