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A Relation between Local and Total Energy in General Relativity

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Abstract. It is shown that, for an asymptotically flat space-time, there exists a collection of conserved vector fields which depend on the local stress energy of the matter and whose integrals over space-like hypersurfaces yield the total ADM or Bondi energy-momentum of the space-time. These vector fields can be used to prove the positivity of the ADM and Bondi energies.

1. Introduction

For an isolated system in general relativity, there are two distinct regimes where the concept of energy is well-defined: Locally, one has the energy density of matter fields given by the stress energy tensor T_{ab}; while asymptotically, far from the system, one can define a total energy of the system which is given by an energy momentum four vector P^a. It has been a long standing problem in general relativity to relate the asymptotically defined total energy to the energy density of matter in the interior. The reason that this problem arises at all, and turns out to be quite difficult to resolve, is that the asymptotic energy and the local energy are related in a complicated nonlinear way through Einstein's field equation. This issue is further complicated by the fact that the asymptotic energy includes contributions from the gravitational field as well as the matter, but there is no local energy density for the gravitational field analogous to T_{ab}.

Actually, there are two distinct notions of "total energy-momentum" for isolated systems. At large space-like separations from the source one can define the Arnowitt-Deser-Misner (ADM) four momentum P_a^{ADM} [1], and at large null separations one can define the Bondi four momentum P_a^{B} [2]. The main difference between the two is that the Bondi momentum is dynamical while the ADM momentum is not, i.e. the Bondi momentum is associated with an instant of retarded time and changes according to the radiation which the source emits,

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