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Analyticity of Effective Coupling and Propagators in Massless Models of Quantum Field Theory

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Abstract. For massless models of quantum field theory, some general theorems are proved concerning the analytic continuation of the renormalization group functions as well as the effective coupling and the propagators. Starting points are analytic properties of the effective coupling and the propagators in the momentum variable k^2, which can be converted into analyticity of β- and γ-functions in the coupling parameter λ. It is shown that the β-function can have branch point singularities related to stationary points of the effective coupling as a function of k^2. The type of these singularities of $\beta(\lambda)$ can be determined explicitly. Examples of possible physical interest are extremal values of the effective coupling at space-like points in the momentum variable, as well as complex conjugate stationary points close to the real k^2-axis. The latter may be related to the sudden transition between weak and strong coupling regimes in quantum chromodynamics. Finally, for the effective coupling and for the propagators, the analytic continuation in both variables k^2 and λ is discussed.

1. Introduction

For massless models of quantum field theory, the renormalization group may be used in order to obtain analytic properties of Green's functions in the coupling parameter λ.[1] It is the purpose of this paper to derive some general theorems concerning the analytic continuation of the effective coupling Λ and of the propagators in the parameter λ and the momentum variable k^2. In particular, we show that the renormalization group function β can have well defined branch points which are associated with stationary points of the effective coupling Λ as a function of k^2. These branch points may be of physical interest in quantum chromodynamics.[2]

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1 For previous work on the analytic continuation in the coupling constant, see [1–5]

2 A preliminary account of this work was given in [5–7]

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