Commun. Math. Phys. 84, 547–556 (1982)

Communications in
Mathematical
Physics

Asymptotic Behaviour of Particle Motion Under Repulsive Forces

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Abstract. The asymptotic nature of motions, as time tends to infinity, is investigated for classical point particles interacting by repulsive two-body potentials U_{ij}. It is found that the conditions $\int_1^\infty U_{ij}(r)dr < \infty$ are necessary and sufficient for asymptotically straight line uniform motion. In the case of equal asymptotic velocities the proof depends only on a certain property of the motion ("partial center of mass convexity") implied by the repulsivity of the potentials.

1. Introduction and Results

This paper continues the previous work of the author [1, 7] and that of Vaserstein [8], as well as Moser [2], Himchenko and Sinai [6] (see also related works of Sinai [4, 5]). Moser [2] considered a system of two particles moving along a line under the action of a strictly repulsive potential U, and he analyzed the connection between the asymptotic properties of the motion in the limit as the time $t \to \infty$, and the potential U. Under certain conditions imposed on U and also supposing that the asymptotic motion is of the form $v_\infty t + c + O(1)$ as $t \to \infty$, an explicit formula for this connection was obtained. In [6] the concept of a *reflectionless* potential is introduced and investigated. This means that for a system of particles moving on a line their motion is asymptotically uniform, i.e., of the form $v_\infty t + c + o(1)$, both when $t \to +\infty$ and when $t \to -\infty$, and the set of asymptotic velocities is the same in the two limits $t \to \pm\infty$, even though the asymptotic velocity of any one particular particle can be different at $-\infty$ and $+\infty$. The theorem is proved that any potential $U(r)$ which satisifies $U(r) \sim C_0 r^{-\alpha}$, $U'(r) \sim C_1 r^{-\alpha-1}$, $U''(r) \sim C_2 r^{-\alpha-2}$ as $r \to \infty$ for some $\alpha > 2$, cannot be reflectionless.

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* The original version of this paper was edited and re-written by A. Lenard from Indiana University. The author expresses his gratitude to Professor Lenard for his work which has significantly improved both the mathematical presentation and the style of the paper

0010-3616/82/0084/0547/$02.00