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The Scaling Limit and Osterwalder–Schrader Axioms for the Two-Dimensional Ising Model

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Abstract. From a Feynman–Kac formula in a Fermion Fock space for the Schwinger functions of the infinite lattice periodic two-dimensional Ising model, scaled and scaling limit Schwinger functions are defined and shown to admit an absolutely convergent series representation. As the critical temperature is attained, it is shown that the scaled Schwinger functions converge and that the resulting scaling limit Schwinger functions obey the Osterwalder–Schrader axioms.

I. Introduction

In [1] the transfer matrix for the two-dimensional finite periodic lattice Ising model was diagonalized in terms of finite lattice Fermions. In [2], starting from a finite lattice Feynman–Kac (F–K) formula, series representations for infinite lattice correlation functions were defined. In [3] we showed that the k-point infinite lattice correlation functions S_k are represented by a F–K formula in a Fermion Fock space. In this representation two sets of canonical Fermion operators, related by a proper linear canonical transformation (plct), are utilized (see [4]) and energy-momentum and spin operators are defined. In [5] a generalization of Wick's theorem was proved for plct and used to obtain explicit series representations for S_k. We also defined series representations for scaling limit Schwinger functions S_k^L from above (T^+) and below (T^-) the critical temperature T_c. The S_k^L are natural candidates for the Schwinger functions of a Wightman field theory.

In this article we show that the S_k^L are the limits of scaled infinite lattice Schwinger functions and that the S_k^L satisfy the Osterwalder–Schrader (O–S) axioms [7].

In Sect. II we introduce scaled Schwinger functions $S_{k\lambda} = S_k(\lambda)/Z_{k\lambda}$, where $\lambda \in [0, 1]$ is a scaling parameter that depends on the temperature T; $\lambda \to 0$ as $T \to T_c$, and $Z_{k\lambda}$ is a wave function renormalization. We prove absolute convergence of the series representation for $S_{k\lambda}$, uniform in λ, as well as convergence to the scaling limit, i.e. $\lim_{\lambda \to 0} S_{k\lambda} = S_k^L$. From these results the series for S_k^L manifestly

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