Commun. Math. Phys. 82, 511–521 (1982)

Communications in Mathematical Physics

Quasi-Free Photon States and the Poincaré Group

W. J. M. A. Hochstenbach

Institut für Theoretische Physik der Universität Göttingen, D-3400 Göttingen, Federal Republic of Germany

Abstract. It is proven that every projectively Poincaré covariant representation of the free photon field defined by a pure quasi-free state is unixarily equivalent to the Fock representation of that field.

1. Introduction

In order to describe photons radiated by a classical external current, coherent and quasi-free states on the quantized free electromagnetic field have been considered by several authors [1–6]. States on the quantized electromagnetic field are, moreover, particularly suited to construct representations of the field that are covariant under a given symmetry group G [3, 7–9[1]]. Quasi-free states also arise in other physical problems, e.g., the vacuum states of generalized free relativistic fields [10], the ground state of the interacting Bose gas at zero temperature [11] and the equilibrium states of the nonrelativistic free Bose gas [12, 13].

Let G be a topological group, $(\mathscr{K}, V)$ a strongly continuous unitary representation of G, E a regular state on the symplectic real-linear space $(\mathscr{K}^r, \sigma)$ induced by the complex Hilbert space $\mathscr{K}$, and $(\mathscr{H}, W, \Omega)$ the Gel'fand-Naimark-Segal (GNS) representation corresponding to E [3]. The Weyl system $(\mathscr{H}, W)$ is called projectively G-covariant if there exists a continuous unitary projective representation $(\mathscr{H}, U)$ of the group G which implements the automorphisms $W(\cdot) \to W(V(s)\cdot)(\forall s \in G)$ of the Weyl system $(\mathscr{H}, W)$. Under additional assumptions for the group G it has been proved in [7, 14] that there exists a bijection from classes of sectors containing coherent states admitting a projectively G-covariant Weyl system, onto the first cohomology group $H^1(G, \mathscr{K}, V)$. In this paper we attempt to extend the foregoing approach to sectors containing quasi-free states.

In Sect. 2 we derive a necessary and sufficient condition for two pure quasi-free states to be in the same sector and we will classify all quasi-free states belonging to

1 During preparation of this paper we received the preprints by Basarab-Horwath, Polley, Reents and Streater dealing with questions treated here

0010-3616/82/0082/0511/$02.20