Commun. Math. Phys. 80, 403–420 (1981)

Communications in Mathematical Physics


# The Geometrical Phase Transitions in a Lattice Model

Koji Kuroda

Institute of Mathematics, University of Tsukuba, Sakuramura
Niiharigun Ibaraki 305, Japan

**Abstract.** We shall consider a finite range model on a square lattice $\mathbf{Z}^3$ and show the existence of bubble, tubular and lamellar phases by estimating the correlation functions at low temperature.

## Introduction

In many systems such as the mixture of water and soap we can observe the phenomena of changes of the geometrical structures according to the density. When we dissolve the soap in water, the system changes from the dissipative state into the state of hexagonal structure, then into the state of lamellar structure and finally into the state of gel. The interaction in the real substances seems too complicated to be analyzed mathematically.

In this paper we shall show that such a phenomenon is realized even in the simple model, though our model may not be the simplest one. Consider the 3-dimensional square lattice $\mathbf{Z}^3$. We arrange oil-particles and water-particles on sites of $\mathbf{Z}^3$, and also arrange soap-molecules on bonds of the lattice. Taking into account the orientational tendency between the hydrophobic group and the hydrophilic group of soap molecules, we set up the interactions between the "components" whose ranges are 1, $\sqrt{2}$, and 2.

We shall prove that the system changes from the state of bubble structure into the tubular structure and finally into the lamellar structure as the density of oil-particles increases by estimating the volume of disordered phase.

We state our results rigorously in Sect. 2 after preparing the necessary definitions in Sect. 1. Section 3 is devoted to the proof of theorems. In this section we consider the correlation functions of Bloch walls and derive several properties of them. As our model does not have simple symmetry in the Ising model, we extend Heilmann's method [3] to obtain the upper bound on the correlation functions.

By using this estimate we obtain the unique solution of the correlation equation, and we also have several properties of correlation functions by refining the Minlos-Sinai method [1, 2].

0010-3616/81/0080/0403/$03.60