Commun. Math. Phys. 80, 381–402 (1981)

Communications in
Mathematical
Physics

A New Proof of the Positive Energy Theorem*

Edward Witten

Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544, USA

Abstract. A new proof is given of the positive energy theorem of classical general relativity. Also, a new proof is given that there are no asymptotically Euclidean gravitational instantons. (These theorems have been proved previously, by a different method, by Schoen and Yau.) The relevance of these results to the stability of Minkowski space is discussed.

I. Introduction

In most classical field theories that play a role in physics, the total energy is the integral of a positive definite energy density T_{00}. This positivity of the energy is usually responsible for ensuring the stability of the ground state.

In gravity, the situation is very different. Even in the weak field case, there is no satisfactory way to define the energy density of the gravitational field. An energy momentum pseudotensor can be defined [1], but it is not a true tensor and is not positive definite. The positivity of the energy in general relativity and the stability of Minkowski space as the ground state are therefore far from obvious.

Although there is no satisfactory way to define the local energy density when gravity is present, one can define the total energy of a gravitating system [2]. The total energy (and momentum and angular momentum) of a gravitating system can be defined in terms of the asymptotic behavior, at large distances, of the gravitational field. However, it is far from obvious that the total energy so defined is always positive.

It is an old conjecture that this total energy is in fact always strictly positive, except for flat Minkowski space, which has zero energy. This matter has been studied by a variety of means.

The energy of a class of gravitational waves was studied by Weber and Wheeler [3]. Positivity of the energy for gravitating systems of special classes was demonstrated by Araki, by Brill, and by Arnowitt, Deser, and Misner [4]. The

---

* Research partially supported by NSF Grant PHY78-01221

0010-3616/81/0080/0381/$04.40