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Renormalization Group and Analyticity in one Dimension: A Proof of Dobrushin's Theorem

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Abstract. We consider one dimensional systems described by many body potentials with finite first moment and prove that the correlation functions are analytic in the interaction parameters. This result is not new (Dobrushin, 1973) but our proof is simpler and physically more transparent. We show that by introducing suitable blocks and averaging over the variables associated to a subset of the blocks (decimation procedure), the resulting effective interaction is such that the system can always be dealt with as a high temperature system.

1. Introduction

In this paper we will deal explicitly with the lattice gas case only; the extension to other cases is considered in Sect. 3.

Consider the elements of $\mathbb{Z}$ as the sites of a one-dimensional lattice, each site may be occupied by 0 or 1 particle. Call K_Λ the product of one copy of the set $\{0,1\}$ for each point of the set $\Lambda \subset \mathbb{Z}$ and $K = \{0,1\}^{\mathbb{Z}}$. $x_\Lambda \in K_\Lambda$ is a configuration of occupied and empty sites in Λ.

The energy of a finite volume Λ, when $\mathbb{Z}\backslash\Lambda$ is empty, is given by:

$$H_\Lambda^\Phi(x_\Lambda) = \sum_{X \subset \Lambda} \Phi(X) \prod_{t \in X} x_t, \tag{1.1}$$

where the potential Φ is a real or complex function defined on the finite subsets of $\mathbb{Z}$. In the sequel we will consider potentials belonging to two Banach spaces $\mathscr{E}$ and $\mathscr{E}'$: $\mathscr{E}$ is the space of translationally invariant real potential with norm:

$$|||\Phi||| = \sum_{X \ni 0} \frac{\operatorname{diam} X + 1}{|X|} |\Phi(X)| < \infty. \tag{1.2}$$

$\mathscr{E}'$ is the larger space of complex potentials with norm

$$|||\Phi||| = \sum_{X \ni 0} \frac{\operatorname{diam} X + 1}{|X|} \sup_{t \in \mathbb{Z}} |\Phi(X+t)|. \tag{1.2'}$$

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