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A Note on the Stability of Phase Diagrams in Lattice Systems

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Abstract. We construct a class of non-symmetry breaking pair interactions, which change the phase diagram of the *n.n.* Ising and classical XY model. Furthermore we improve earlier obtained constraints on the decrease of interactions, necessary to get analyticity properties of the pressure in manifolds of non-symmetry breaking interactions.

1. Introduction, Notation and Some Known Results

Heuristically, in thermodynamics one expects that the Gibbs phase rule holds, in the sense that there are manifolds in some interaction space on which
1. suitable thermodynamic functions are analytic
2. the number of possible phases remains the same [4].

In [1] it was shown that in spaces $\mathscr{B}_g$ defined by

$$\|\Phi\|_g = \sum_{0 \in X \subset \mathbb{Z}^d} g(X)|\Phi(X)|$$

2. does not hold if $g(X)$ is only dependent on the number of points in X and
1. does not hold if $g(X)$ increases more slowly than $(diam(X))^{1/2}$.

The existence of manifolds on which more than one phase coexist has been shown by Peierls contour arguments. In finite dimensional interaction spaces, containing classical interactions of finite range, this has been done in [2]; for a more restricted class of interactions, but in the infinite dimensional subspace of pair interactions with $g(X) = diam(X)$, in [3].

In this paper we improve the results of [1] for the 2 dimensional Ising model. We construct a classical long range perturbation, which does not break the symmetry of the transition, but changes the phase diagram when added to the Ising interaction with any positive strength. This implies that the result in [3] in a sense is best possible. For analyticity we give the improved condition (which is necessary but probably not sufficient in view of our result on the stability of the phase diagram) that $g(X)$ has to increase at least as $diam(X)^{2/3}$.

Furthermore we consider the broken rotation symmetry of the classical XY model and show that in that case the stability of the phase transition is lower than in the case of a broken discrete symmetry.

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