Commun. Math. Phys. 77, 111–126 (1980)

Communications in Mathematical Physics

Correlation Inequalities and the Decay of Correlations in Ferromagnets*

Barry Simon
Departments of Mathematics and Physics, Princeton University, Princeton, NJ 08544, USA

Abstract. We prove a variety of new correlation inequalities which bound intermediate distance correlations from below by long distance correlations. Typical is the following which holds for spin 1/2 nearest neighbor Ising ferromagnets:

$$\langle s_\alpha s_\gamma \rangle \leqq \sum_{\delta \in B} \langle s_\alpha s_\delta \rangle \langle s_\delta s_\gamma \rangle$$

where B is any subset of the lattice whose removal divides the lattice into pieces with α, γ in distinct components. We describe various applications, e.g. the above inequality implies the critical exponent inequality $\eta < 1$.

1. Introduction

This paper originated in my attempt to understand some results announced by Dobrushin [7] in the summer of 1979. Dobrushin considers a class of model systems including lattice gas models. Given two bounded regions $\Lambda \subset \Lambda'$, he considers the variation of a Gibbs state restricted to Λ as spins exterior to Λ' are varied. Among other results, he proved that if this dependence falls off as the inverse of a sufficiently high power of $d(\partial\Lambda, \partial\Lambda')$, then it automatically falls exponentially. It was this kind of result that I wanted to understand. We will deal with the related result that falloff of the two point function at a sufficiently fast inverse power rate implies exponential decay. We will accomplish this by proving various new correlation inequalities. Typical of the results we will prove is:

Theorem 1.1. *Let $\langle \sigma_\alpha \sigma_\gamma \rangle$ denote the two point function of a spin 1/2 nearest neighbor (infinite volume, free boundary condition) Ising ferromagnet at at some fixed temperature. Fix α, γ and B, a set of spins whose removal breaks the lattice in such a way that α and γ lie in distinct components. Then:*

$$\langle \sigma_\alpha \sigma_\gamma \rangle \leqq \sum_{\delta \in B} \langle \sigma_\alpha \sigma_\delta \rangle \langle \sigma_\delta \sigma_\gamma \rangle . \tag{1.1}$$

* Research partially supported by USNSF under Grant MCS-78-01885

0010-3616/80/0077/0111/$03.20