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Fibre Bundles and Supersymmetries

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Abstract. By extending the frame bundle of a manifold admitting a spin structure to a principal fibre bundle with group the super-Poincaré group we are able to derive the supergravity Lagrangian and its invariances under supersymmetry and other transformations in a global form.

1. Introduction

Since geometric formulations have proved so successful in modern physics it was natural that once the concept of supersymmetries had been accepted it would be attempted to recast the component formulations [1] in terms of a more geometric language. So far the closest way to achieve this aim has been the idea of superspace [2]: the extension of space-time to include four extra fermionic dimensions together with the differential geometry implied by the increased structure.

However, this approach suffers from various defects. First it seems that to agree with the results derived by more direct methods it is necessary to abandon the obvious generalization of the Levi-Civita connection and impose an ad hoc torsion condition [3]. Second, the theory includes a large number of extraneous fields whose interpretation poses difficulties. Third, no great simplification in terms of concepts required or algebra needing to be done is achieved. Lastly, the transformation used in superspace mix up the bosonic and fermionic variables leading to difficulties in interpretation and in identification of the theory with the seemingly bosonic space-time we actually inhabit.

In this paper we present a formulation of super-gravity in terms of fibre bundles [4] which we claim overcomes the objections to the concept of superspace. By considering a fibre bundle whose base space is space-time no difficulty with nilpotent co-ordinates arises, the transformations corresponding to those in superspace always acting in another space. The only fields arising are the vierbein e_μ^a and the spin 3/2 field ψ_μ^a, with the connection ω_μ^{ab} being given by an obvious generalisation of the normal Levi-Civita ω_μ^{ab}. The supergravity Lagrangian and its invariance under local supersymmetry transformations can be quite easily derived.

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