Commun. Math. Phys. 76, 177–209 (1980)

Communications in
Mathematical
Physics

Finite Total Cross-Sections in Nonrelativistic Quantum Mechanics

V. Enss[1,★] and B. Simon[2,★★]

[1] Institute for Advanced Study, Princeton, New Jersey 08540, USA
[2] Departments of Mathematics and Physics, Princeton University, Princeton, New Jersey 08544, USA

Abstract. We present a simple geometric method for estimating total cross-sections in two-body and more generally two cluster scattering. We discuss a variety of aspects of total cross-sections including large coupling constant behavior.

1. Introduction

The total scattering cross-section is one of the basic objects in a quantum scattering problem. Nevertheless, there has been relatively little rigorous study of it until recently and, even in the physics literature, there appears to be no discussion of some very basic questions. For example, let $\sigma(\mathbf{k};V)$ denote the total cross-section for scattering involving the pair $(-\frac{1}{2}\Delta, -\frac{1}{2}\Delta+V)$ at incident momentum $\mathbf{k}$. Except [37], we know of no study of the large g behavior of $\sigma(\mathbf{k};gV)$. Here we will study this question $\left(\text{or more precisely, the growth of } \int_a^b \sigma(k\mathbf{e};gV)dk\right)$ and obtain a bound (see Sect. 4) by $Cg^{(\nu-1)/(\alpha-1)}$ if $V\sim|x|^{-\alpha}$ at infinity in ν dimensions with

$$\alpha>\tfrac{1}{2}(\nu+1). \tag{1.1}$$

[(1.1) is needed for σ to be finite.] In some special cases, we will obtain a lower bound with the same power behavior (see Appendix 2).

A second question which we mention explicitly is the finiteness of the total cross-section (including scattering into charged clusters or more than two clusters) for Coulomb scattering with two cluster initial state with both clusters neutral. Again, we know of no previous results on this problem, although Combes has informed us that he and Tip [12] have obtained similar results with different

★ On leave from Dept. of Physics, University of Bielefeld, Federal Republic of Germany; address after April 1, 1980: Dept. of Mathematics, University of Bochum, D-4630 Bochum 1, Federal Republic of Germany

★★ Research partially aupported by USNSF under Grant MCS-78-01885

0010-3616/80/0076/0177/$06.60