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Statistical Fluid Dynamics: Unstable Fingers

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Abstract. This paper is the first in a series by the authors devoted to the study of fingers in fluid surfaces. Fingers are a form of surface instability which occur on many length scales. In particular, they may occur on length scales small relative to the natural dimensions of the problem; in this sense the instability is similar to turbulance. In the present study, the transition from stability to instability is determined by a critical value in a viscosity ratio. This series of papers is devoted to methods of accurate numerical computation. We find that the random choice method gives excellent resolution of fingered surfaces and discontinuities. Even an unstable interface, with three to four well developed fingers can be resolved on a coarse grid of 10 to 15 zones wide.

1. Introduction

Fingers are a form of surface instability; they occur in a number of physical phenomena. They often occur with a characteristic length (finger width, or finger spacing) which is small relative to the natural dimensions of the problem. In fact, fully developed fingering is approximately scale invariant, in that it may occur on all length scales from the dimensions of the problem down to a cutoff length provided by some dissipative mechanism.

We study this phenomenon numerically in the context of two phase flow in porous media.

The equations of two phase immiscible flow in porous media have the form

$$\frac{\partial s}{\partial t}+\nabla\cdot(\mathbf{v}f(s))=0\,, \qquad \mathbf{v}=-k(s)\nabla p\,, \tag{1.1}$$

$$\nabla\cdot\mathbf{v}=0\,, \tag{1.2}$$

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