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The Navier-Stokes Equations on a Bounded Domain*

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Abstract. Suppose U is an open bounded subset of 3-space such that the boundary of U has Lebesgue measure zero. Then for any initial condition with finite kinetic energy we can find a global (i.e. for all time) weak solution u to the time dependent Navier-Stokes equations of incompressible fluid flow in U such that the curl of u is continuous outside a locally closed set whose 5/3 dimensional Hausdorff measure is finite.

1. Introduction

Definition 1.1. Suppose f is a C^∞ function defined on an open subset V of $R^3 \times R$. If $i \in \{1,2,3\}$ then $D_i f$ is the partial derivative of f with respect to the i component of R^3. The partial derivative of f with respect to the R component of $R^3 \times R$ is denoted by $D_t f$. The letter t is used because the second component of $R^3 \times R$ represents time. The vector function $(D_1 f, D_2 f, D_3 f)$ is written Df. The function Δf is defined on the set V by $(\Delta f)(x,t) = \sum_{i=1}^{3} D_i(D_i f)(x,t)$. When the range of f is R^3 we define the functions $\operatorname{div}(f): V \to R$ and $\operatorname{curl}(f): V \to R^3$ by

$$(\operatorname{div}(f))(x,t) = \sum_{i=1}^{3} D_i f_i(x,t)$$

and

$$(\operatorname{curl}(f))(x,t) = ((D_2 f_3 - D_3 f_2)(x,t), (D_3 f_1 - D_1 f_3)(x,t), (D_1 f_2 - D_2 f_1)(x,t)).$$

We extend these definitions in the obvious way to the case where f is a distribution. Hausdorff measure is defined in Definition 6.5, R^+ is the set $\{t \in R : t > 0\}$, L^p is the Lebesgue space of p-integrable functions with norm $\| \ \|_p$, and the summation convention for repeated indices is used. If A and B are sets then $A \sim B = \{x \in A : x \notin B\}$.

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