Commun. Math. Phys. 70, 243–269 (1979)

Communications in Mathematical Physics

Surface Tension and Phase Transition for Lattice Systems

J. R. Fontaine[1] and Ch. Gruber[2]

[1] Institut de Physique Théorique, Université Catholique de Louvain, Louvain-la-Neuve, Belgique
[2] Laboratoire de Physique Théorique, EPFL, Lausanne, Switzerland

Abstract. We introduce the surface tension for arbitrary spin systems and study its general properties. In particular we show that for a large class of systems, the surface tension is zero at high temperature. We also derive a geometrical condition for the surface tension to be zero at all temperature. For discrete spin systems this condition becomes a criterion to establish the existence of a phase transition associated with surface tension. This criterion is illustrated on several examples.

1. Introduction

The phenomenon of "phase transition" is one of the important problems of statistical mechanics because of its physical and mathematical interest. As is well known several definitions have been introduced to discuss the existence of phase transitions and the equivalence of these definitions has not always been established. It seems possible however to classify all phase transitions into two classes, those which occur with a spontaneous breakdown of the symmetry group of the system (coexistence of several phases, existence of local order parameters) and those which occur without any symmetry breakdown[1].

One of the standard methods to prove the existence of a phase transition for lattice systems is the "Peierls argument"; its generality relies on the fact that it takes explicitly into account the underlying group structure of the system [1, 7]. However, it is well adapted for systems which have a complete breakdown of the internal symmetry group at low temperature and does not apply as readily to describe phase transition associated with partial symmetry breakdown.

In this article, we propose to introduce *the surface tension as definition of phase transition*, i.e., we shall say that "there exists a phase transition associated with a

1 In such cases we know that there exist phase transitions associated with the coexistence of several phases and the existence of a local order parameter; however it is *not* always so: there exist models for which the Gibbs state is unique at all temperatures and which do exhibit a phase transition; there exist also models which show a phase transition without any local order parameter

0010-3616/79/0070/0243/$05.40