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Group Duality and the Kubo-Martin-Schwinger Condition

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Abstract. We consider clustering G-invariant states of a C^*-algebra $\mathfrak{A}$ endowed with an action of a locally compact abelian group G. Denoting as usual by F_{AB}, G_{AB}, the corresponding two-point functions, we give criteria for the fulfillment of the KMS condition (w.r.t. some one-parameter subgroup of G) based upon the existence of a closable map T such that $TF_{AB}=G_{AB}$ for all $A, B\in\mathfrak{A}$. Closability is either in $L^\infty(G)$, $B(G)$, or $\mathscr{C}_\infty(G)$, according to clustering assumptions. Our criteria originate from the combination of duality results for the group G (phrased in terms of functions systems), with density results for the two-point functions.

1. Introduction

The so-called Kubo-Martin-Schwinger (KMS) condition plays an important role both in physics, where it is the modern expression of the "Gibbs structure" (independant of the thermodynamic limit) [1] and in the theory of von Neumann algebras where separating normal states possess this property w.r.t. their "modular automorphism groups" [2]. With $\mathfrak{A}$ a C^*-algebra and $t\to\alpha_t$ a one-parameter automorphism group of $\mathfrak{A}$, a state ω is called β-KMS for α whenever, to each pair $A, B\in\mathfrak{A}$, there is a function f of the complex variable, continuous and bounded in the strip $0\leqq \operatorname{Im} z\leqq\beta$, holomorphic in its interior, with boundary values

$$\begin{cases} F_{AB}(t) = \omega(B\alpha_t(A)) = f(t) \\ G_{AB}(t) = \omega(a_t(A)B) = f(\beta t + i) \end{cases}, \qquad t\in\mathbb{R}. \tag{1.1}$$

This condition can alternatively be stated as follows in terms of Fourier transforms (tempered distributions)

$$\hat{F}_{AB}(p) = e^{-\beta p}\hat{G}_{AB}(p), \qquad p\in\mathbb{R}. \tag{1.2}$$

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