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Lack of Screening in the Continuous Dipole Systems*

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Abstract. We study continuous statistical systems interacting via a regularized dipole potential in the grand canonical ensemble. In the explicitly given region of high temperature (or low density) we show that the effective potential between two parallel dipoles is *not* absolutely integrable (it is, however, square integrable), which implies that the effective potential does not fall off faster than $|x|^{-3}$ in some directions.

We consider continuous statistical systems interaction via a dipole potential in the grand canonical ensemble. Since the dipole potential is not stable, we regularize the potential at the origin so that the system is well-defined. In [3] we have shown that the thermodynamic limit of correlation functions and the pressure exists. It is of great interest to know whether there exists some kind of screening. By defining an effective potential between two dipole moments in the grand canonical ensemble it turns out that the effective potential between two parallel dipoles is not absolutely integrable in the explicitly given region of high temperature (or low density). On the other hand, we will show that it is square integrable. The result implies that the effective potential does not fall off faster than $|x|^{-3}$ in some directions (for high temperatures or low densities). This is in contrast to dilute coulomb gas for which Debye exponential screening occurs [1].

Since the dipole potential is of long-range, the method of the cluster expansion [7] cannot be applied. The technique used in the proof is the Gaussian integral formalism of statistical mechanics, which has already proved to be a very powerful technique [2, 3, 4, 6]. We primarily follow the notation of [3] and use some of its results.

We first briefly introduce the notation and the model we are interested in. The *regularized* dipole potential between a particle with dipole moment $\sigma \in \mathbb{R}^3$, at position $x \in \mathbb{R}^3$ and one with dipole moment σ' at position x' is defined by

$$V(\sigma, x; \sigma', x') = (2\pi)^{-3/2} \int d^3k e^{ik\cdot(x-x')}(\sigma\cdot k)(\sigma'\cdot k)k^{-2}|\hat{\kappa}(k)|^2 \tag{1}$$

where $\hat{\kappa}(k)$ is the Fourier transform of a regularization function $\kappa(x)$ in $C_0^\infty(\mathbb{R}^3)$

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