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The Local Central Limit Theorem for a Gibbs Random Field

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Abstract. We extend the validity of the implication of a local limit theorem from an integral one. Our extension eliminates the finite range assumption present in the previous works by using the cluster expansion to analyze the contribution from the tail of the potential.

1. Definitions and Results

1.1. Assumptions

We consider a ν-dimensional lattice spin system, to each $x \in \mathbb{Z}^\nu$ is associated a spin s_x that can take all the integer values lying between the two integers n, m. We denote by σ the $\max(|n|, |m|)$, A *configuration* s_Λ in a subset $\Lambda, \Lambda \subset \mathbb{Z}^\nu$ is given by an element of $\{n, \dots m\}^\Lambda$. If Λ, M are two disjoint subsets of $\mathbb{Z}^\nu$, we denote by $s_\Lambda \vee s_M$ the spin configuration in $\Lambda \cup M$ individuated by s_Λ, s_M. The *interaction* is given by a pair long range potential of the form $J(x-y)s_x s_y$ where J is a real function on $\mathbb{Z}^\nu$. We assume that:

$$\sum_{t \in \mathbb{Z}^\nu} |J(t)|^{1/2} = \gamma < \infty \tag{1.1}$$

1.2. Definition. Let Λ be a finite subset of $\mathbb{Z}^\nu$. The *Gibbs conditional distribution* on the set of configurations in Λ, with condition $s_{\mathbb{Z}^\nu \setminus \Lambda}$ is defined as:

$$p_\Lambda(s_\Lambda | s_{\mathbb{Z}^\nu \setminus \Lambda}) = \exp\left\{ \sum_{\substack{x, y \in \Lambda \\ x \neq y}} J(x-y)s_x s_y + \sum_{x \in \Lambda} h_x(s_x | s_{\mathbb{Z}^\nu \setminus \Lambda}) \right\} / Z_\Lambda(h, J)$$

where

$$h_x(s_x | s_{\mathbb{Z}^\nu \setminus \Lambda}) = \sum_{y \in \mathbb{Z}^\nu \setminus \Lambda} J(x-y)s_x s_y + J(0)s_x^2 \tag{1.3}$$

and $Z_\Lambda(h, J)$ is the partition function for the set of spins in Λ, with pair interaction J,