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Non-Translation Invariant Gibbs States with Coexisting Phases III: Analyticity Properties*

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Abstract. We consider the spatially inhomogeneous Gibbs states for the three dimensional Ising and Widom-Rowlinson models. We prove the analyticity in $z = \exp(-2\beta J)$ for small $|z|$ of the spin correlation functions of these Gibbs states and of the surface tension.

1. Introduction

We consider the three-dimensional Ising model, with nearest neighbor ferromagnetic interactions, in a box with $\pm$ boundary conditions (*b.c.*); the spins surrounding the top (bottom) half of the box are equal to $+1$ (-1). It is known [1, 2] that, at low temperatures, these *b.c.* lead, in the thermodynamic limit, to an extremal non-translation invariant Gibbs state whose correlation functions cluster exponentially.

In this paper we prove that these correlation functions are analytic in the variable $z = \exp(-2\beta J)$ for $|z|$ sufficiently small; β is the inverse temperature and J is the coupling. We also study the surface tension τ, i.e., the thermodynamic limit of the difference in free energy per unit cross-section between the system with $\pm$ *b.c.* and one with $+$ *b.c.* (all spins on the boundary equal to $+1$). We show that $\tau - 2\beta J$ is analytic in z at low temperatures. Moreover, we extend our results to Widom-Rowlinson models on a lattice; the systems discussed in the first two papers of this series [3, 4].

The method we use is a low-temperature expansion of the Minlos-Sinai or Kirkwood-Salzburg type. (This is similar to the method used by Gallavotti in his work [5] on the two-dimensional Ising model with $\pm$ *b.c.*) The Minlos-Sinai equations [6] were originally developed for studying the low-temperature behavior of the pure phase, i.e., the state obtained with the $+$ *b.c.* The configurations of the system with $+$ *b.c.* are described by means of contours and Minlos and Sinai apply Kirkwood-Salzburg equations to the 'gas' of contours. The factor $\exp(-2\beta J)$ plays the role of an activity.

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