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A New Method in the Combinatorics of the Topological Expansion

D. Bessis
Service de Physique Théorique, CEN Saclay, F-91190 Gif-sur-Yvette, France

Abstract. We reduce the counting problem for the vacuum diagrams of a ϕ^4 theory to a moment problem. As a consequence we are able to give the generating function for the counting of diagrams on a torus with one hole, besides the known result for planar diagrams. The method can be extended to ϕ^n theory and also to the counting of diagrams on a torus with an arbitrary number of holes.

I. Introduction

In their paper Planar Diagrams [1], E. Brézin, C. Itzykson, C. Parisi and J.B. Zuber have discussed the combinatorics of Quartic Vertices, and found the generating function $E^{(0)}(g)$ which solves the counting problem for the vacuum diagrams in the planar approximation. The technic used was the saddle point method. Unfortunately this method does not provide an easy way to reach even the next generating function $E^{(1)}(g)$ which solves the counting problem on a torus with one hole.

In this paper, we have obtained for this generating function a rather simple expression:

$$E^{(1)}(g) = \tfrac{1}{12}\log(2-a^2), \tag{I.1}$$

with, following the notation in [1]:

$$12ga^4 + a^2 - 1 = 0, \tag{I.2}$$

where the root to be taken (I.2) is the root regular at $g=0$. Of course we have also verified that the generating function $E^{(0)}(g)$ is given by:

$$E^{(0)}(g) = -\tfrac{1}{2}\log a^2 + \tfrac{1}{24}(a^2-1)(9-a^2). \tag{I.3}$$

The general case, the computation of $E^{(k)}(g)$ for $k \geqq 2$ will be considered elsewhere.

As shown in [1], the generating functions $E^{(k)}(g)$ appear as coefficients in the asymptotic expansion:

$$-\frac{1}{n^2}\log\frac{I_n(g/n)}{I_n(0)} = E_0(g) + \frac{E_1(g)}{n^2} + \frac{E_2(g)}{n^4} + \dots \tag{I.4}$$

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