Commun. Math. Phys. 69, 89–98 (1979)

Communications in
Mathematical
Physics

Boundedness of Total Cross-Sections in Potential Scattering

A. Martin

CERN, Geneva, Switzerland

Abstract. If a spherically symmetric potential belongs to the Rollnik class, i.e., if

$$I=\frac{1}{(4\pi)^2}\int\frac{|V(x)|\,|V(x')|}{|x-x'|^2}\,d^3x\,d^3x'$$

is finite, the total cross-section is finite, and an explicit bound on this quantity can be given in terms of I. We also investigate the case of non-spherically symmetric potentials, and show that if I is less than unity, the total cross-section averaged over the directions of the incident beam at a given energy is finite.

1. Introduction

Recently the question of boundedness of total cross-sections in potential scattering has been re-examined by Amrein and Pearson [1]. These authors have shown that if one can be satisfied by statements of the form: "the cross-section is finite for almost all energies" it is no longer necessary to assume local regularity of the potential as was done in some previous work on the subject [2]. Crudely speaking, all the authors [1, 2] find that the cross-sections are finite if the potential decreases sonewhat faster than $|x|^{-2}$ at large distances (in the case of three space dimensions).

Here we want to use a method which, in its essence, is not new, and was applied 15 years ago to get bounds on the scattering amplitude itself [3]. This method is inspired by the work of Froissart to obtain bounds on elementary particle scattering amplitudes [4]. The general idea is that the partial wave amplitudes and cross-sections for small angular momentum can be bounded by unitarity, while for large angular momentum the Born approximation (or nearest singularity contribution in the case of elementary particles!) gives a reasonable estimate.

It is not very difficult to see that the total cross-section, for spherically symmetric potentials, calculated in the Born approximation, will be finite at all energies, except possibly at threshold if the potential belongs to the Rollnik class [5], such that

$$I=\frac{1}{(4\pi)^2}\int\frac{|V(x)|\,|V(x')|}{|x-x'|^2}\,d^3x\,d^3x' \tag{1.1}$$