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An Existence Theorem for Multimeron Solutions to Classical Yang-Mills Field Equations*

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Abstract. In this paper we prove the existence of solutions to a class of boundary value problems for a singular nonlinear elliptic partial differential equation in a half plane. By a recent paper of J. Glimm and A. Jaffe, this proves the existence of multimeron solutions to the classical SU(2) Yang-Mills field equations in Euclidean space.

I. Introduction and Results

In this paper we prove existence of solutions to a class of boundary value problems for the singular elliptic equation

$$r^2 \Delta \psi = \psi^3 - \psi \tag{1.1}$$

in the half plane $\mathbb{R}^2_+ = \{(r,t) \in \mathbb{R}^2 | r > 0\}$, where $\Delta = \frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial t^2}$. We remark that $r^2\Delta$ is the Laplace-Beltrami operator for the Poincaré halfplane [$\mathbb{R}^2_+$ with the metric $r^{-2}(dr^2 + dt^2)$]. The boundary values are specified by an increasing sequence $\{t_i | 1 \leq i \leq 2n\}$ of real numbers and the requirement

$$\begin{aligned} &\lim_{r \to 0} \psi(r,t) = (-1)^i, \qquad t_i < t < t_{i+1}, \qquad i = 0, \dots, 2n, \\ &\lim_{(r,t) \to \infty} \psi(r,t) = 1, \end{aligned} \tag{1.2}$$

where we have defined $t_0 = -\infty$, $t_{2n+1} = \infty$. Our principal result is

Theorem 1.1. *There is a function Ψ, real analytic and satisfying* (1.1) *in* $\mathbb{R}^2_+$, *which assumes the boundary values* (1.2).

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