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Exponential Clustering for Long-Range Integer-Spin Systems*

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Abstract. By using Kirkwood-Salsburg equations for classical spin systems with unbounded integer values we prove exponential decay (resp. power law decay) for exponential (resp. power law) decaying potentials. We use these results to prove the mass gap in the two-dimensional Higgs-Villain model in the weak coupling region.

1. Introduction

In a previous paper [1] we have worked out a Kirkwood-Salsburg equation for unbounded integer spin systems whose Hamiltonians are positive definite quadratic forms and investigated the conditions under which it led to a unique equilibrium state expressed as a convergent series in powers of the "Kirkwood-Salsburg" operator. We applied these results to the two-dimensional Higgs-Villain model, which is reduced to such a system by duality transformations [2–6].

In the present paper we use the series expansion of the equilibrium state to prove exponential (resp. power law) decay of truncated correlation functions when the interaction potential decays exponentially (resp. with a power law). Similar results for lattice gas and continuum systems are obtained in [7] and [8]. In fact, our method, especially in Theorem 2, is very similar to that of [7]. The results of [8] apply to very general systems, but only with finite range interactions.

We apply our results once more to the two-dimensional lattice Higgs-Villain model to prove that in a certain range of parameters (g^2/T large and g^2 small) truncated expectations of local observables exhibit exponential decay, i.e. there is a mass gap. Moreover our lower bound on the mass approaches the "bare mass" $gqT^{-1/2}$ as $T\to 0$ with g^2/T fixed. There is some question whether this mass gap should be interpreted as a Higgs mechanism. A massive photon would be associated with a Yukawa type potential between external charges, contradicting

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