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Spectral Properties of Certain Composition Operators Arising in Statistical Mechanics

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Abstract. By applying the theory of linear positive operators in a Banach space we derive spectral properties of certain composition operators in the Banach space $A_\infty(\Omega)$ of holomorphic functions over some domain $\Omega \subset \mathbb{C}^n$. Examples of such operators are provided by the so called generalized transfer matrices of classical one-dimensional lattice systems.

Introduction

It was shown in a series of papers [1–5] how one can define generalized transfer matrices for one-dimensional classical lattice systems even if the interaction between spins on different lattice sites is not of finite range. These transfer matrices are given as abstract linear trace class operators acting in Banach spaces of holomorphic functions of finite or infinite many complex variables. The general form of such a transfer matrix T is given as follows

$$Tf(z) = \sum_{k=1}^{m} \varphi_k(z) f \circ \psi_k(z). \tag{1}$$

In (1) the functions φ_k and f are holomorphic in the variable $z \in \Omega$, where Ω is some open bounded region in $\mathbb{C}^n$ or in some complex Banach space B. The ψ_k's are holomorphic mappings of Ω into itself, such that T defines a linear operator in the Banach space $A_\infty(\Omega)$ of holomorphic functions over Ω. It was shown in [5] that under certain conditions on the mappings ψ_k the operator T as defined in (1) defines a nuclear operator of order zero which insures that T and all powers T^n, $n \geqq 1$, have a trace.

In statistical mechanics one knows that the physical properties of a system which allows for a transfer matrix T are determined by the spectral properties of this operator [6]. For the one-dimensional systems for which a transfer matrix as in (1) exists one knows from different methods that all thermodynamic potentials are analytic functions in all parameters of interest, like for instance the temperature. This reflects the fact that such a system does not have a phase transition.

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