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Imperfect Bifurcation in the Presence of Symmetry

M. Golubitsky[1]* and D. Schaeffer[2]**

[1] Department of Mathematics, Arizona State University, Tempe, Arizona 85281, USA
[2] Department of Mathematics, Duke University, Durham, North Carolina 27706, USA

Introduction

Consider the familiar principle that typically (or generically) a system of m scalar equations in n variables where $m>n$ has no solutions. This principle can be reformulated geometrically as follows. If S is a submanifold of a manifold X with codimension m (i.e. $m=\dim X-\dim S$) and if $f:\mathbf{R}^n\to X$ is a smooth mapping where $m>n$, then usually – or generically – Image $f\cap S$ is empty. One of the basic tenets in the application of singularity theory is that this principle holds in a general way in function spaces. In the next few paragraphs we shall try to explain this more general situation as well as to explain its relevance to bifurcation problems.

First we describe an example through which these ideas may be understood. Consider the buckling of an Euler column. Let λ denote the applied load and x denote the maximum deflection of the column. After an application of the Lyapunov-Schmidt procedure the potential energy function V for this system may be written as a function of x and λ alone and hence the steady-state configurations of the column may be found by solving

$$(0.1)\quad G(x,\lambda)=\frac{\partial V}{\partial x}(x,\lambda)=0\,.$$

See for example [6, Sect. 6]. It is shown there that near the buckling point (which we assume to be at $\lambda=0$) we may write

$$(0.2)\quad G(x,\lambda)=x^3-\lambda x+\dots\,.$$

Moreover, the lowest order terms dominate so that the pitchfork $x^3-\lambda x=0$ describes qualitatively the various steady-state configurations of the column near the buckling point.

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