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On the Break-down of Completeness of Wave Operators in Potential Scattering

D. R. Yafaev*

St. Banach International Mathematical Center, Warsaw, Poland, and
Leningrad Department of Mathematical Institute, Leningrad, USSR

Abstract. We study the Schrödinger operator with a potential that vanishes at infinity but the rate of falloff of the potential depends on the direction. It turns out that for such potentials scattering theory becomes in general multichannel.

1. Introduction

It is well-known that the continuous spectrum of the self-adjoint Schrödinger operator $H = -\Delta + q$ in the space $\mathscr{H} = L_2(\mathbb{R}^m)$ coincides with the half-line $[0, \infty)$ if $q(x) \to 0$ when $|x| \to \infty$. Moreover, if the estimate

$$|q(x)| \leqq C(1+|x|)^{-1-\varepsilon}, \qquad \varepsilon > 0, \tag{1.1}$$

is satisfied, then for the pair $H_0 = -\Delta$, H the wave operators (WO) exist and are complete [1]. In constructing the scattering theory the estimate (1.1) is in some sense optimal: if $q(x) = c|x|^{-1}$ the WO do not exist. On the other hand, in the theory of multiparticle scattering [2–4] when the potential does not decrease in some directions the WO may exist but may not be complete. The break-down of completeness is connected with the existence of eigenvalues of operators describing subsystems of smaller numbers of particles; thus the continuous spectrum of the operator H covers the half-line $[\varkappa, \infty)$ where $\varkappa < 0$.

This paper is devoted to the study of the Schrödinger operator with a potential that tends to 0 if $|x| \to \infty$, but the rate of falloff depends on the direction. It appears that for such potentials scattering theory becomes, in general, multichannel. This means that although the WO W may exist, its range $R(W)$ need not coincide with the whole absolute continuous subspace $\mathscr{H}_{ac}$ of the operator H. The break-down of completeness of the WO is thereby connected with the following situation, corresponding to the existence of negative eigenvalues for subsystems for multiparticle Schrödinger operators.

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* Permanent address: Leningrad Department of Mathematical Institute, USSR Academy of Science, 27, r. Fontanka, 191011 Leningrad, USSR