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On the Asymptotic Behaviour of Infinite Gradient Systems

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Abstract. We consider gradient systems of infinitely many particles in one-dimensional space interacting via a positive invariant pair potential Φ with a hard core. The main assumption is that Φ is strictly convex within the range R of Φ (where R is a fixed number $\leqq \infty$). Under some technical conditions we prove the following theorems: Let the initial distribution be given by a translation invariant point process ϱ on $\mathbb{R}^1$. Then there exists only one extreme equilibrium state ϱ with a given intensity $I(\varrho)$ satisfying $I(\varrho) \geqq R^{-1}$, and all ergodic initial distributions ϱ with an intensity $I(\varrho) \geqq R^{-1}$ converge weakly as $t \to \infty$ to the extreme equilibrium state with the same intensity.

1. Introduction

In classical statistical mechanics one considers configurations of many particles, which in the mathematical idealization means infinitely many particles, moving according to Newton's equations

$$(1.1) \quad \ddot{x}_i(t) = - \sum_{j \neq i} \operatorname{grad} \Phi(x_i(t) - x_j(t))$$

with $i \in \mathbb{N}$, $x_i \in \mathbb{R}^d (d \in \mathbb{N})$ and a pair potential Φ. Only recently [1] has a non-equilibrium existence proof for (1.1) in the case $d = 1, 2$ been found, and in the series of papers [4] the equilibrium states are characterized as Gibbs measures corresponding to the potential Φ. However the problem of the asymptotic behaviour of the system (1.1) is not understood as yet (apart from some cases with a degenerate potential Φ there are no results as yet). For a survey on the present state see [2].

Related to (1.1) is the system of stochastic equations

$$(1.2) \quad dx_i(t) = \left[- \sum_{j \neq i} \operatorname{grad} \Phi(x_i(t) - x_j(t)) \right] dt + \beta^{-1/2} d\omega_i(t) \quad (i \in \mathbb{N})$$

with independent Wiener processes $\omega_i(t)$ and the inverse temperature $\beta (\beta > 0)$. As follows from [9] the Gibbs measures for the potential $2\beta\Phi$ are equilibrium states