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Confinement of Static Quarks in Two Dimensional Lattice Gauge Theories

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Abstract. In two dimensional Higgs models on a lattice with Abelian or Nonabelian compact gauge group, fractional static charges are always confined.

1. Introduction

We consider lattice gauge theories [1] in two Euclidean space time dimensions with compact gauge group G with nontrivial center.[1] We admit multiplets of Bose fields $\phi(x)$, but no fermions. The Bose fields are required to transform trivially under an abelian (finite or Lie-) subgroup Γ contained in the center of G. Let C a closed path, $U[C] \in G$ the parallel transporter around C, and χ the character of a representation of G which is not trivial on Γ. We show that the Wilson loop integral satisfies

$$\langle \chi(U[C]) \rangle \leqq \chi(\mathbb{1}) \exp\{-\alpha \cdot (\text{area enclosed by } C)\} \tag{1.1}$$

with $\alpha > 0$. This indicates confinement of fractional static charges. Fractional means with nontrivial transformation law under Γ.

Example. $G = SU(2), \Gamma \simeq \mathbb{Z}_2, \phi$ any (reducible or irreducible) multiplet of scalar fields with integral "colour" isospins, $\chi(U) = \operatorname{tr} U$.

The proof of this result is elementary. It is essential that the Euclidean Lagrangean is real. Coupling constants are otherwise arbitrary.

For the special case of the Abelian Higgs-Villain model [9] a stronger result than ours is known [2] for small coupling constant β^{-1}.

2. An Example

For the sake of clarity we consider first a model with gauge group $G = SU(2)$, $\Gamma = \{\pm \mathbb{1}\} \simeq Z_2$, ϕ scalar fields transforming according to a one valued repre-

[1] By definition, the center of G consists of those elements γ of G which commute with all elements, i.e. $\gamma g = g\gamma$ for all g in G. Among the simply connected simple Lie groups, the requirement of nontrivial center excludes the exceptional groups G_2, F_4 and E_8