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Vacuum Energy in $g\varphi_d^4$-Theory for $g \to \infty$

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Abstract. In the nonlocal $g\varphi_d^4 (d \geqq 1)$ and local $g\varphi_2^4$ theory the S-matrix is obtained in a form of the functional integral which is proved to exist. The density of vacuum energy

$$E(g) = -\lim_{V \to \infty} \frac{1}{V} \ln \langle 0|S_V(g)|0\rangle$$

is investigated. It is proved to be analytic through the whole complex g-plane except for the negative real axis and point $g=0$. Its asymptotic behaviour for $g \to \infty$ is found.

1. Statement of the Problem

The φ^4-theory is rather popular; and quite a number of papers are devoted to investigations of various aspects of this theoretical model. Without exaggeration, one can say that it is just this model that tests majority of theoretical methods and approaches. It is difficult to report all contributions of investigations of different aspects of the φ^4-theory.

In the given paper, the $g\varphi^4$-theory is applied to study the density of vacuum energy

$$E(g) = -\lim_{V \to \infty} \frac{1}{V} \ln \langle 0|S_V(g, \varphi)|0\rangle . \tag{1.1}$$

The function $E(g)$ in (1.1) is finite in the local φ_d^4-theory for space-time dimensions $d=1$ (anharmonic oscillator) and $d=2$ (the so called φ_2^4-theory) only. For $d>2$ the function $E(g)$ does not exist at all in the local theory because of ultraviolet divergences. However, $E(g)$ is finite in the nonlocal theory when the causal Green functions $\tilde{D}(k^2)$ of the scalar field φ decreases rather fast in the Euclidean direction $(k^2 = k_0^2 - \mathbf{k}^2 \to -\infty)$.

In the given paper we will consider all these cases and obtain the analytical properties in the complex g-plane and the asymptotical behaviour of the function $E(g)$ for $g \to \infty$.

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