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Dilation Analyticity in Constant Electric Field

I. The Two Body Problem*

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Abstract. The resolvent of the operator $H_0(\varepsilon, \theta) = -\Delta e^{-2\theta} + \varepsilon x_1 e^{\theta}$ is not analytic in θ for θ in a neighborhood of a real point, if the electric field ε is non-zero. (One manifestation of this singular behavior is that for $0 < |\mathrm{Im}\,\theta| < \pi/3$, $H_0(\varepsilon, \theta)$ has no spectrum in the finite plane.) Nevertheless it is shown that the techniques of dilation analyticity still can be used to discuss the long-lived states (resonances) of a system described by a Hamiltonian of the form $H = -\Delta + \varepsilon x_1 + V(x)$.

I. Introduction

It is interesting that two of the first problems which arose in the early days of quantum mechanics, the Stark and Zeeman effects, have until recently remained largely unstudied from a mathematical point of view (and to some extent, for large fields at least, also from a physical point of view). Notable exceptions are contained in the work of Titchmarsh [33] and Riddell [29] on the Stark effect in hydrogen. More recent rigorous work on the Zeeman effect can be found in [6–10, 24] and on the Stark effect in [5, 16, 17, 34], however many interesting questions remain to be answered.

It is the purpose of this paper to discuss the long-lived states or resonances associated with systems described by Hamiltonians of the form $H = -\Delta + \varepsilon x_1 + V$ in $L^2(\mathbb{R}^n)$ (Stark Hamiltonians), where in this paper V is a multiplication operator which in some sense vanishes at ∞. In addition we wish to lay the groundwork for a study of the N-body Stark problem (e.g. atomic systems with N electrons). The resonances we will discuss are not solely associated with the operator H [21, 22, 23]. This can be understood by noting the fact that for a large class of potentials, V, (including $V(x) = -Z/|x|$ in $L^2(\mathbb{R}^3)$) H is for each $\varepsilon > 0$ unitarily equivalent to multiplication by x_1 [17]:

$$H = U x_1 U^{-1}$$

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