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Quark Confinement in the Two-Dimensional Lattice Higgs-Villain Model*

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Abstract. We prove quark confinement in the two-dimensional lattice Higgs-Villain model in the weak coupling region by using a Kirkwood-Salsburg equations for unbounded spins.

0. Introduction

In [1] Polyakov pointed out the role that instantons might play in the problem of confinement in gauge theories by showing that in the $U(1)$ three-dimensional pure gauge theory in the weak coupling region, due to the instantons' effect, external charges are confined.

His argument also predicts charge confinement in the Abelian two-dimensional Higgs model because of the presence there of the Nielson-Olessen vortices. The Wilson loop expectation value [2] has indeed been computed in this model [3, 4] by taking into account only the instantons' contribution to the path integral in the dilute gas approximation. The answer for the quark-antiquark potential is the typical one-instanton contribution $\sim e^{-1/g^2}$ where g is the gauge coupling constant.

We want to test these ideas on lattice gauge theories, which combine the advantage of a rigorous formulation with many of the features of the continuum theories. We consider therefore the two-dimensional Higgs-Villain model [5–8], which differs from the usual $x-y$ version adopted for lattice scalar electrodynamics by the substitution

$$e^{-\frac{1}{T}\cos\theta} \to \sum_{m\in\mathbb{Z}} e^{-\frac{1}{2T}(\theta-2\pi m)^2} .$$

In the weak coupling region, the one we are going to investigate ($T\to 0$), the above two functions are practically identical. Nevertheless the latter version is

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