Commun. math. Phys. 63, 49–96 (1978)

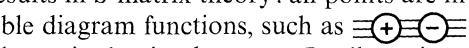
Communications in Mathematical Physics

The $u=0$ Structure Theorem

D. Iagolnitzer

CERN, CH-1211 Geneva, Switzerland, and
Service de Physique Théorique, CEN Saclay, F-91190 Gif-sur-Yvette, France

Abstract. The previous theorem of the author on the analytic structure of the bubble diagram functions that occur in unitary equations (and are kernels of products of connected scattering operators $S^c_{m,n}$ or $(S^{-1})^c_{m,n}$, and related quantities), is extended to a class of situations, called here in general $u=0$ points, that were not covered by this earlier result.

This new theorem, which is proved on the basis of a refined macrocausality condition, resolves one of the remaining crucial problems in the derivation of discontinuity formulae and related results in S-matrix theory: all points are in fact $u=0$ points for some of the bubble diagram functions, such as  ($\equiv(S^{-1})^c_{3,3}S^c_{3,3}$), that are encountered even in the simplest cases. In all previous approaches, ad hoc technical assumptions with no a priori physical basis were required for these terms.

The origin of the $u=0$ problem is the absence of information, in general, on a product of distributions that are boundary values of analytic functions from opposite directions, and more generally on the essential support, or singular spectrum, of a product of distributions whose essential supports contain opposite directions. On the other hand, the recent results obtained by Kashiwara-Kawai-Stapp in the framework of hyperfunction theory apply mainly to phase-space factors, whose bubbles are constants times conservation δ-functions rather than actual scattering operators. The present work has basically required the development of new physical and mathematical ideas and methods. In particular, a new general result on the essential support of a product of bounded operators is presented in $u=0$ situations, under a general regularity property on individual terms. The latter follows in the application from the refined macrocausality condition, in the same time as information on the essential support of S-matrix kernels.

0010-3616/78/0063/0049/$09.60