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Conditional Equilibrium and the Equivalence of Microcanonical and Grandcanonical Ensembles in the Thermodynamic Limit

Michael Aizenman[1]*, Sheldon Goldstein[2]**, and Joel L. Lebowitz[2]***

[1] Department of Physics, Princeton University, Princeton, New Jersey 08540, USA
[2] Department of Mathematics, Rutgers University, New Brunswick, New Jersey 08903, USA

Abstract. Equivalence (allowing for convex combinations) of microcanonical, canonical and grandcanonical ensembles for states of classical systems is established under very mild assumptions on the limiting state. We introduce the notion of conditional equilibrium (C.E.), a property of states of infinite systems which characterizes convex combinations of limits of microcanonical ensembles. It is shown that C.E. states are, under quite general conditions, mixtures of Gibbs states.

1. Introduction

Systems of infinite spatial extent [1–3] offer mathematically convenient idealizations of macroscopic equilibrium systems. The statistical mechanical theory of such systems may be obtained either by considering the thermodynamic (infinite volume) limit of finite systems described by appropriate Gibbs ensembles (e.g., micro-canonical, canonical, grand-canonical, pressure) or by considering equilibrium states of infinite systems directly. While the first route is the more physical, the latter is mathematically more direct and can often provide useful insights into the phenomena for which the large size (on a molecular scale) of macroscopic systems plays an essential role, e.g., phase-transitions. In addition the formal theory of infinite systems may offer useful mathematical tools for the study of local phenomena in macroscopic systems. Various results valid in the thermodynamic limit can be formulated as simple properties of the infinite system. It is for these reasons that the infinite system point of view is often adopted.

In the infinite system formalism, equilibrium macroscopic systems are commonly represented by Gibbs states, or measures, on the space of locally finite configurations [2, 3]. These are defined by the DLR equations [2, 3] which are satisfied by the limits of finite volume grand-canonical ensembles. One may also define "conditional equilibrium" states (C.E.) by similar equations which are

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