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On the Discrete Spectrum of the N-Body Quantum Mechanical Hamiltonian. I

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Abstract. We discuss N-body kinematics and study the Berezin-Sigal equations in configuration space. Assuming that the threshold of the continuous spectrum is zero and that the pair potentials satisfy $|V(x)| \leqq C(1+|x|^2)^{-\varrho}$, $x \in \mathbb{R}^3$, $\varrho > 1$ (together with some technical hypotheses), we show that the discrete spectrum of the hamiltonian in the center of mass system is finite. The case of negative threshold will be treated in a further publication.

1. Introduction

The basic theorem on the quantum mechanical hamiltonian in the center of mass system, due to Hunziker [5], van Winter [14], and Zhislin [19], states that under suitable assumptions on the potentials the essential spectrum of this hamiltonian consists of a half-line $[\mu, \infty)$, $\mu \leqq 0$, while the discrete spectrum lies below μ and its only possible accumulation point is μ itself. This immediately suggests the question of determining conditions for finiteness or infinitude of the discrete spectrum. This problem has been attacked successfully by several authors under various conditions on the potentials. Zhislin [19] has shown that atoms have infinite discrete spectrum. Simon [12] proved that if the potentials decay as $|x|^{-2+\delta}$, $\delta \geqq 0$, at infinity then the discrete spectrum may be infinite. In the three body case conditions for finiteness and/or infinitude have been obtained by Combescure and Ginibre [3], Iorio [6], Yafaev [15–17]. The N-body case was analysed by Sigal [10], Yafaev [18], Simon [13] (using geometrical methods). In this article we prove finiteness of the discrete spectrum in case $\mu = 0$ for potentials falling-off as $|x|^{-2-\delta}$, $\delta > 0$, (together with some technical assumptions; see Section 5), using the Berezin-Sigal equations, and working entirely in configuration space. The following notation and definitions will be used throughout this work. If $\mathfrak{X}$ and $\mathfrak{Y}$ are Banach spaces we denote by $B(\mathfrak{X}, \mathfrak{Y})$ [resp. $B_0(\mathfrak{X}, \mathfrak{Y})$] the set of all bounded (resp. compact) operators from $\mathfrak{X}$ to $\mathfrak{Y}$. In case $\mathfrak{X} = \mathfrak{Y}$ we write simply $B(\mathfrak{X})$ and

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